Systematic translation of Cryptographic Axioms by Bi-deduction
(Work in progress)

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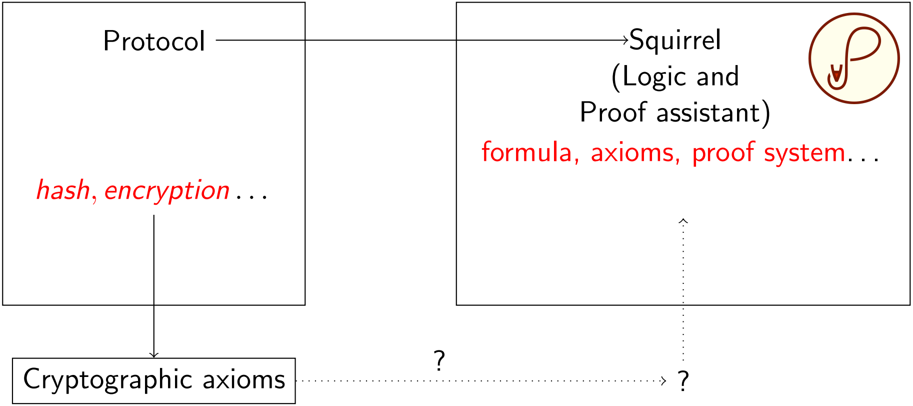
Context

Formal verification of cryptographic protocols.

Examples:

- Secure online payment (authentication)
- Secure messaging (privacy)

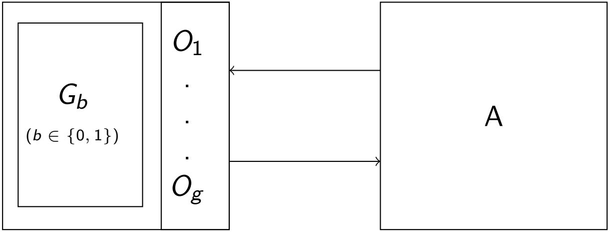
Squirrel and Cryptographic axioms



This talk

- Insight of cryptographic axioms and Squirrel logic
- Capture cryptographic axioms into formulas, while ensuring their correctness.

Cryptographic axiom



Definition (Indinstinguishability)

For any polynomial-time and randomized algorithms A,

$$|\Pr(A^{G_0} = 1) - \Pr(A^{G_1} = 1)|$$

is negligible (*i.e.*, roughtly exponentially small in the length of the keys).

Example: PRF games

Intuition: a pseudo random function is a function that “seams” random.

Example (PRF games)

Game G_0

$Challenge(x)$:

$return\ \boxed{\mathsf{h}(\mathsf{x},\mathsf{k})}$

Game G_1

$Challenge(x)$:

sample(r)

$return\ \boxed{\mathsf{r}}$

Example: PRF games

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Example (PRF games)

Game G_0 $\quad$ *Init* :

sample(k);

Challenge(x) :

return $\boxed{h(x,k)}$

Game G_1 $\quad$ *Init* :

sample(k);

Challenge(x) :

sample(r)

return $\boxed{r}$

Example: PRF games

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Example (PRF games)

Game G_0	Init : sample(k);	$\mathit{Hash}(x)$: *return* $\mathsf{h}(x, \mathsf{k})$	$\mathit{Challenge}(x)$: *return* $\boxed{\mathsf{h}(\mathsf{x},\mathsf{k})}$
Game G_1	Init : sample(k);	$\mathit{Hash}(x)$: *return* $\mathsf{h}(x, \mathsf{k})$	$\mathit{Challenge}(x)$: sample(r) *return* $\boxed{\mathsf{r}}$

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Example (PRF games)

Game G_0	$Init$:	$Hash(x)$:	$Challenge(x)$:
	sample(k);	$L := x :: L$	sample(r)
	$L := []$;	$return\ \mathsf{h}(x, \mathsf{k})$	$if\ x \notin L$
			$L := x :: L$;
			$return\ \boxed{\mathsf{h}(\mathsf{x},\mathsf{k})}$

Game G_1	$Init$:	$Hash(x)$:	$Challenge(x)$:
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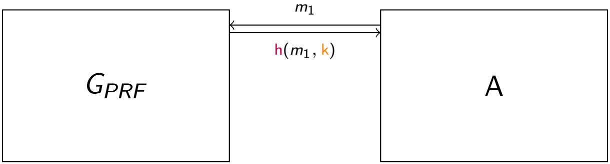
Example: PRF games

Example (PRF pair of games)

Game G_{PRF}

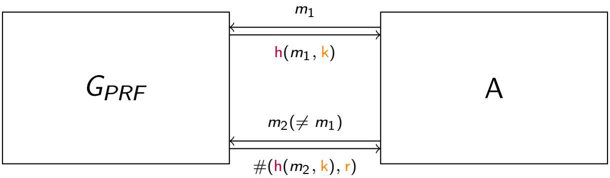
$Init$:	$Hash(x)$:	$Challenge(x)$:
sample(k);	$L := x :: L$	sample(r)
$l := []$;	$\mathsf{h}(x, \mathsf{k})$	$if\ x \notin L$
		$L := x :: L$;
		$\boxed{\#(\mathsf{h}(\mathsf{x},\mathsf{k}),\mathsf{r})}$

Playing with PRF: sequence of messages



$$m_1, h(m_1, k)$$

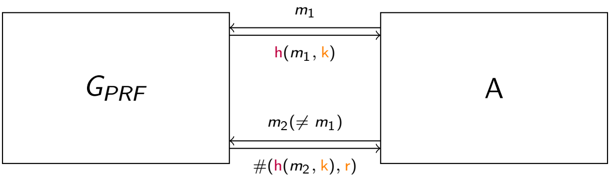
Playing with PRF: sequence of messages



$$m_1, h(m_1, k), m_2, \#(h(m_2, k), r)$$

$$:= (\ (m_1, h(m_1, k), m_2, h(m_2, k)),\\ (m_1, h(m_1, k), m_2, r)\ )$$

Playing with PRF: sequence of messages



$$\mathrm{equiv}((m_1, \mathrm{h}(m_1, \mathrm{k}), m_2, \#(\mathrm{h}(m_2, \mathrm{k}), \mathrm{r})))$$

It there exists an adversary that can distinguish between this two sequences of messages, then PRF doesn't holds.

Terms and formulas

Definition (Terms)

Intuition: terms represents messages

Semantics: interpreted as computation of a turing machine.

$$
\begin{aligned}
t := &\mid \mathsf{r} && \text{(sampling)}\\
&\mid f(t_1, \ldots, t_n) && \text{(function application)}\\
&\mid \#(t_0, t_1) && \text{(left/right difference)}
\end{aligned}
$$

Definition (Equivalence formulas)

$$\mathsf{equiv}(\vec{t})$$

PRF axiom schema

Question : is this formula always valid according to PRF ?

$$\mathsf{equiv}((m_1, \mathsf{h}(m_1, \mathsf{k}), m_2, \#(\mathsf{h}(m_2, \mathsf{k}), \mathsf{r})))$$

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- $m_1 = \mathsf{k}$: k adversary must not directly access the key.

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Definition (PRF axiom schema)

For all terms $\vec{t}$ and m, samplings k and r such that

- k never appears in $\vec{t}$ and m except: $h(\_, k)$.
- for all subterms $h(m', k)$ of $\vec{t}$ or m: m and m' are never equal.
- r never appears in $\vec{t}$ and m

$$\frac{}{\text{equiv}((\vec{t}, \#(h(m, k), r)))}$$

Problems and contributions

Problem with this method

Ad-hoc and manual work for each cryptographic axioms:

- Axiom schema design
- Correctness proof (understand the logic and its semantics)
- Implementation (understand the code)

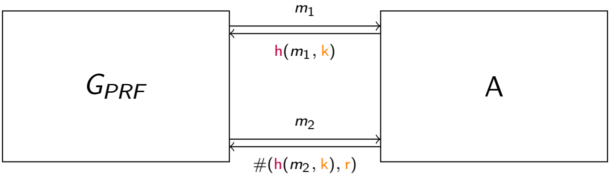
Contributions

- A systematic way to prove that a formula is a consequence of a cryptographic axioms.
- Automation (WIP)

Changing point of view

Input :

$$m_1, h(m_1, k), m_2, h(m_1, k), m_3, \#(h(m_3, k), r_{fresh})$$



Question: Does there exists such A ?

Bi-deduction

Construction of bi-deduction judgement: starting point

Intuition: there exists A such that $A^{G_{PRF}}() = \vec{v}$.

$$\triangleright \vec{v}$$

Definition (Link between Bi-deduction and Equivalence)

Intuition: if an adversary can compute $\vec{v}$ then the formula equiv$(\vec{v})$ holds.

$$\frac{\triangleright \vec{v}}{\mathsf{equiv}(\vec{v})}\ \textsc{Bi-deduction}$$

Proof system

Goal: Proof system for this bi-deduction judgement.

What can compute an adversary?

- An adversary is a program: function applications
- An adversary can draw samples: samplings.
- Interaction with the games: oracles calls

Proof system

Goal: Proof system for this bi-deduction judgement.

What can compute an adversary?

- An adversary is a program: function applications (done)
- An adversary can draw samples: samplings.
- Interaction with the games: oracles calls

Definition

Function application inference rule

$$\mathrm{FA}\;\frac{\triangleright \vec{t}}{\triangleright f(\vec{t})}$$

Samplings

Example

$$\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{h}(\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{k})$$

adversary game

We need to keep track of the owner of each sampling.

Definition (Tags)

$$Tag = \{ T_a, T_{g,0}, T_{g,1}, \dots \}$$

$$\mathsf{n} \leftarrow T_a$$
$$\mathsf{s} \leftarrow T_a$$
$$\mathsf{k} \leftarrow T_{g,key}$$

Extending bi-deduction with constraints

Adding sampling tagging

C records who sampled what:

$$C : \triangleright \vec{v}$$

Definition (Adversary samplings)

$$\textsc{Adv sampling}\ \frac{C : \triangleright \vec{v}}{C, < \mathsf{n}, T_a >: \triangleright \mathsf{n}, \vec{v}}$$

$$\frac{\overline{\emptyset : \triangleright \emptyset}}{}$$

$$: \triangleright \mathsf{h}(\mathsf{n}, \mathsf{s})$$

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$$\dfrac{\dfrac{\dfrac{}{\emptyset : \rhd \emptyset}}{< \mathsf{s}, T_a >: \rhd \mathsf{s}}\textsc{Adv sampling}}{\dfrac{< \mathsf{n}, T_a >, < \mathsf{s}, T_a >: \rhd \mathsf{n}, \mathsf{s}}{: \rhd \mathsf{h}(\mathsf{n}, \mathsf{s})}}\textsc{Adv sampling}$$

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Oracle calls on example

Definition (Oracle rule : instenciated for hash oracle)

$$\textsc{Hash}\ \frac{C : \triangleright m, \vec{v}}{C, < \mathsf{k}, T_{g,key} >: \triangleright \mathsf{h}(m, \mathsf{k}), \vec{v}}$$

Example

$$\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{h}(\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{k})$$

$$\dfrac{\dfrac{*}{C : \triangleright \mathsf{h}(\mathsf{n}, \mathsf{s})}}{}$$

Oracle calls on example

Definition (Oracle rule : instenciated for hash oracle)

$$\textsc{Hash}\quad \frac{C : \rhd m, \vec{v}}{C, < \mathsf{k}, T_{g,key} >: \rhd \mathsf{h}(m, \mathsf{k}), \vec{v}}$$

Example

$$\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{h}(\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{k})$$

$$\frac{\dfrac{*}{C : \rhd \mathsf{h}(\mathsf{n}, \mathsf{s})}}{C, < \mathsf{k}, T_{g,key} >: \rhd \mathsf{h}(\mathsf{h}(\mathsf{n}, \mathsf{s}), \mathsf{k})}\,\textsc{Hash}$$

Consistency of tagging

$$\frac{\textsc{Bi-deduction}\quad C : \triangleright \vec{t} \qquad \vdash \mathit{Valid}(C)}{\mathrm{equiv}(\vec{t})}$$

$\mathit{Valid}(C)$ ensures :

- Not two samples for one "role" ($e.g.$, $k \leftarrow T_{g,key}, k' \leftarrow T_{g,key}$)
- No sample owned by both the adversary and the oracles

Oracle rule : Challenge

Oracle rule instenciated for challenge

$$\frac{C : \triangleright m, \vec{v}}{C, < \mathsf{r}, T_g >, < \mathsf{k}, T_{g,key} >: \triangleright \#(\mathsf{h}(m, \mathsf{k}), \mathsf{r}), \vec{v}}$$

$$m \longrightarrow \mathsf{h}(m, \mathsf{k}) \longrightarrow \#(\mathsf{h}(m, \mathsf{k}), \mathsf{r})$$

Oracle rule : Challenge

Oracle rule instenciated for challenge

$$\frac{C : \rhd m, \vec{v}}{C, < \mathsf{r}, T_g >, < \mathsf{k}, T_{g,key} >: \rhd \#(\mathsf{h}(m, \mathsf{k}), \mathsf{r}), \vec{v}}$$

$$m \xrightarrow{\hspace{3cm}} \mathsf{h}(m, \mathsf{k}) \xrightarrow{\hspace{2cm}} \#(\mathsf{h}(m, \mathsf{k}), \mathsf{r})$$

$L = [m]$ $\quad\quad\quad$ $L = [m, m]$?

Hoare's style oracle triplets

Definition (Hoare's triples for an oracle o)

Let ϕ and ψ be pre and post conditions.

$$\{\phi\}c_o[\vec{t},\vec{s}]\{\psi\}$$

is correct iff:

- When ϕ holds the oracle o return $c_o[\vec{t},\vec{s}]$ on input $\vec{t}$ and samplings $\vec{s}$.
- ψ holds after o call.

When $L = lis \implies m \notin L$ then

$$\{L := lis\}\#(\mathsf{h}(m,\mathsf{k}),\mathsf{r})\{L := m :: lis\}$$

is correct.

Adding pre and post conditions

$$\phi, \psi; C : \triangleright \vec{v}$$

Oracle rule

Definition (Oracle rule)

$$\textsc{Oracle}\ \frac{\phi, \psi; C : \triangleright \vec{t}, \vec{v} \qquad \{\psi\} c_o[\vec{s}, \vec{t}]\{\theta\}}{\phi, \theta; C, < \vec{s}, T_{g,\dots} >: \triangleright c_o[\vec{s}, \vec{t}], \vec{v}}$$

Induction and abstract semantics

Chalenges following:

Recursive terms : induction in the proof system

$\rightarrow$ Approximate pre and post condition

- Abstract representation of the game memory.
- Adapt the rule and/or the proof of soundness ragarding these approximation.

Conclusion

- Formal framework linking games, adversaries, and formulas
- Bi-deduction judgment to capture adversaries interacting with a game
- Proof system for this judgment
- Over-approximation of pre and post conditions (WIP)
- Implementation
  - Automation of proof search (WIP)
  - How to generate/check Hoare triples? (Future Work)

Questions ?

Gergei Bana and Hubert Comon-Lundh, A computationally complete symbolic attacker for equivalence properties, Proceedings of the 2014 ACM SIGSAC Conference on Computer and Communications Security, Scottsdale, AZ, USA, November 3-7, 2014 (Gail-Joon Ahn, Moti Yung, and Ninghui Li, eds.), ACM, 2014, pp. 609–620.

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