

# Unrestricting restrictions in ProVerif

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GT FMS

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# Symbolic (Dolev-Yao) models

The attacker can...



Read / Write



Intercept

But they do not...



Break cryptography



Use side channels

Concurrent systems where dishonest parties have complete control over network communication  
**but** cryptography is idealised

# Automated Verification

**Proverif**

**Two main verifiers**

**Tamarin**

Large class of cryptographic primitives

Reachability and equivalence properties

Unbounded number of sessions

**Interactive interface**

**Push-button**

Lemmas  
Axioms  
Restrictions

**Lot of guidance**

**Almost no guidance**

**Less automated**

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**More guidance  
[BCC-S&P22]**

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**Lot of guidance**

**Less automated**

**More automation  
[CDDK-JCS22]**

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# Lemmas / Axioms / Restrictions

## Lemmas

Intermediary property useful to prove the main query

Proved by the tool

## Axioms

Similarly to Lemmas but assumed by the tool

## Restrictions

Restricts the search space of traces on which to prove the main query

Sometimes useful to avoid heavy encoding

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# Lemmas / Axioms / Restrictions in ProVerif [BCC22]

## Already useful

Basis of GSVerif tool

Allow to handle stateful protocols

Add precision to ProVerif

Instrumental in the verification of TLS-ECH, Voting protocols, ZCash

```
axiom st:bitstring, x:bitstring, y:bitstring;  
event(precise(st,x)) && event(precise(st,y)) ==> x = y.
```

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## Lemmas are correspondence queries

Premise can be any predicate (events, attacker, mess, table, user-defined)

$$F_1 \wedge \dots \wedge F_n \Rightarrow \phi$$

Disjunctions and conjunction of events, inequalities, equalities, and disequalities

$$\underbrace{\forall x_1, x_2, \dots, x_k . F_1 \wedge \dots \wedge F_n}_{\text{Variables in the premises}} \Rightarrow \underbrace{\exists y_1, \dots, y_\ell . \phi}_{\text{Remaining variables in } \phi}$$

Variables in the premises

Remaining variables in  $\phi$

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Allow to be more expressive in the order of events

```
axiom id:voter, v,v':vote, i,j:time;  
event(hasVoted(id,v))@i && event(hasVoted(id,v'))@j ==> i = j.
```

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Allow to be more expressive in the order of events

Allow to be more efficient in the application of the lemma

```
lemma i,j:nat;  
  event(A(i)) && event(B(j)) && i < j ==> event(C(i,j)).
```

instead of

```
lemma i,j:nat;  
  event(A(i)) && event(B(j)) ==>  
  i >= j || ( i < j && event(C(i,j)) ).
```

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Allow to be more expressive in the order of events

Allow to be more efficient in the application of the lemma

Avoid non-termination scenarios

```
lemma id:voter, v:vote, i,j:time;  
event(VoteCounted(id,v))@i ==> attacker(v)@j && i > j.
```

$$s\text{-event}(\text{VoteCounted}(id, v)) \wedge H \rightarrow \text{att}(v)$$

Clause generated when the vote is revealed by the tally

Removed by application of the lemma



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Does not allow disequalities and inequalities in the premises

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**Allow to prove properties on complex data structure**

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**Allow to prove properties on complex data structure**

**Allow to prove liveness and accountability properties**

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# Liveness properties [BDKK-EuroSnP17]

## Local progress

Processes need to be reduced as far as possible, that is until they wait for a message

## Resilient channels

Messages sent on resilient channel must be delivered

## External non-determinism

Any process  $P + Q$  reduces to  $R$  if  $P$  or  $Q$  reduces to  $R$ .

**All these properties can be enforced by  
"forward" restrictions**

# Liveness properties [BDKK-EuroS&P17]

## External non-determinism

Any process  $P + Q$  reduces to  $R$  if  $P$  or  $Q$  reduces to  $R$ .

All these properties can be enforced by  
"forward" restrictions

$P = a; P'$

$Q = b; Q'$

$P + Q$

translated into

$\text{event } B; (\text{event } M; a; \text{event } E; P' \mid \text{event } M; b; \text{event } E; Q')$

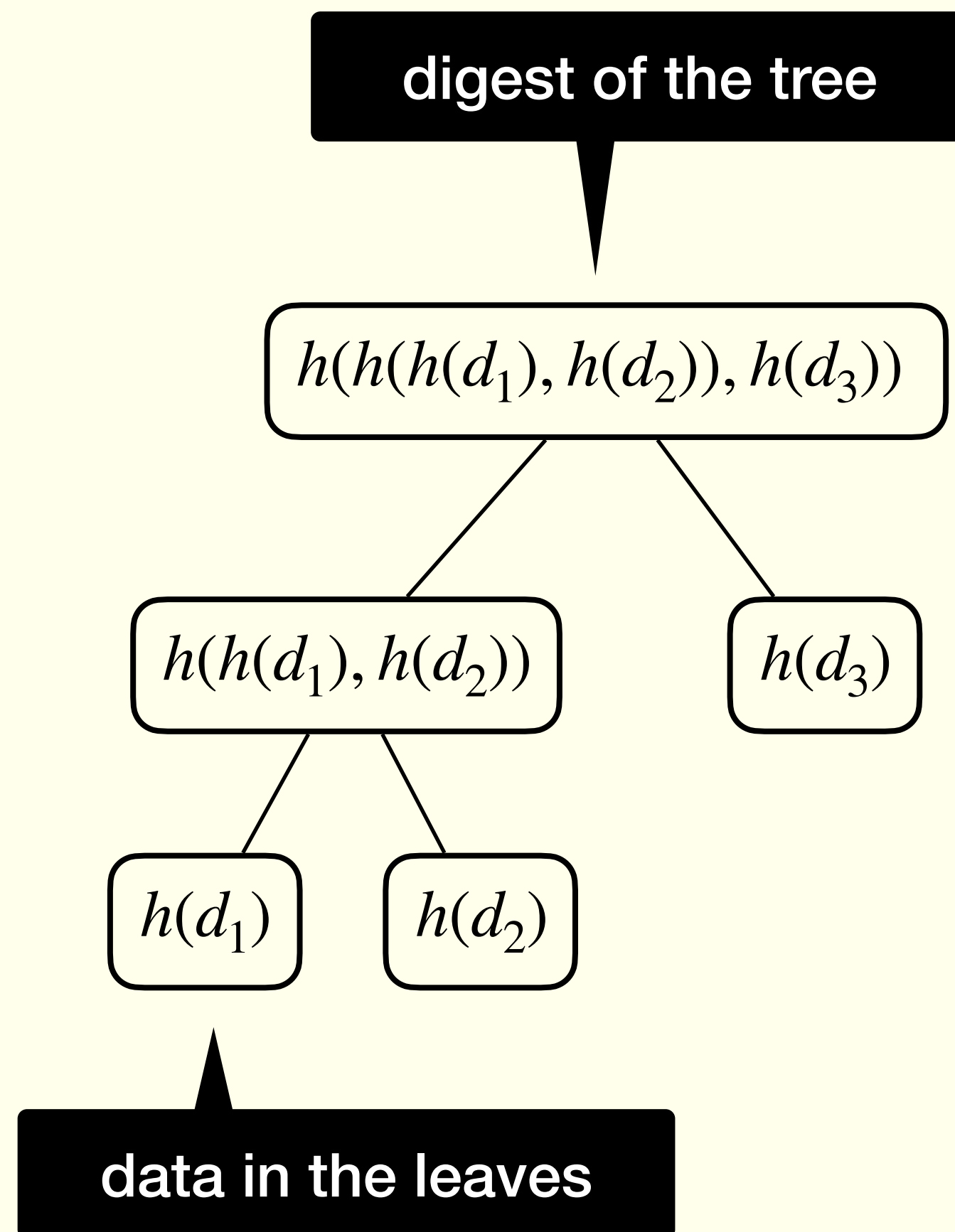
restriction

$\text{event}(B) \implies \text{event}(E);$

$\text{event}(M)@i \ \&\& \ \text{event}(M)@j \implies i = j.$

# Properties on complex data structures [CMR-EuroS&P23]

## Merkle trees



Good to model ledgers

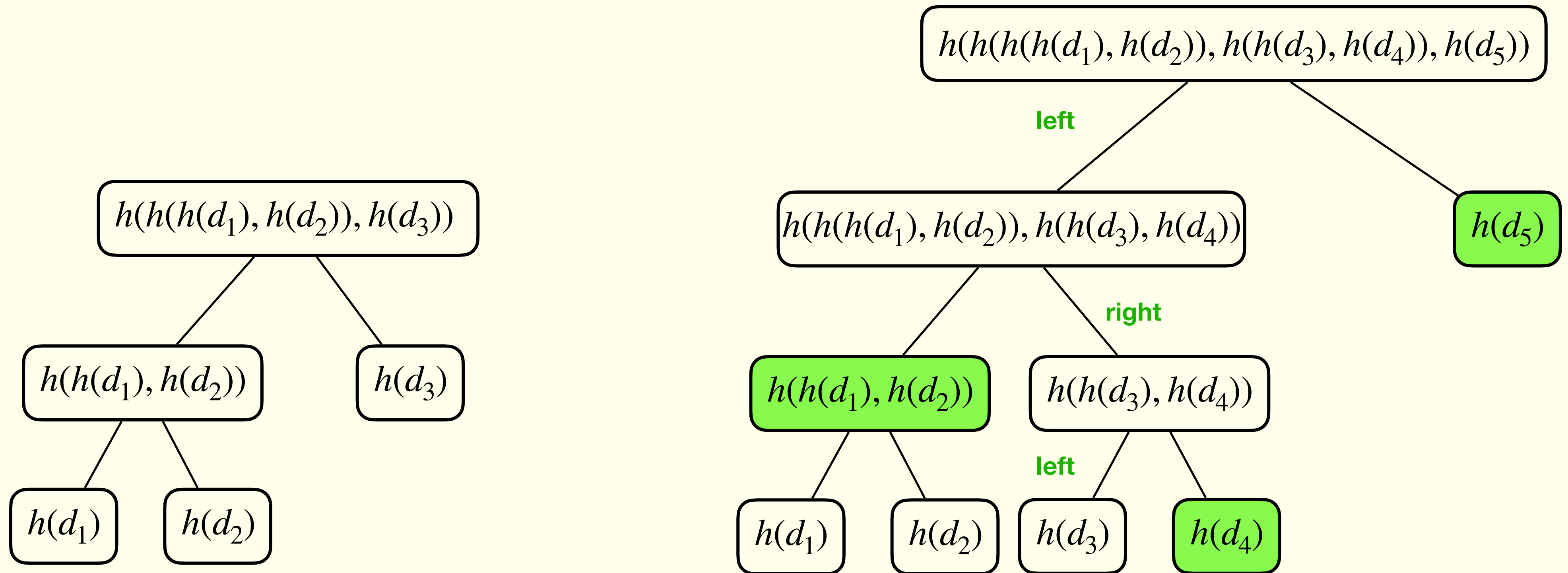
Append only structure

Proof of presence in  $O(\log(n))$

Proof of extension in  $O(\log(n))$

# Properties on complex data structures [CMR-EuroS&P23]

## Merkle trees



In green, proof of extension between the two trees

# Properties on complex data structures [CMR-EuroS&P23]

## Defining verification predicates through Horn clauses

```
(* Proof of presence *)  
  
fun PP(list):proof_of_presence [data].  
  
clauses  
  forall x:bitstring;  
    verify_pp(PP(nil),x,hash(leaf(x)));  
  forall pl:list, x:bitstring, d_left,d_right:digest;  
    verify_pp(PP(pl),x,d_left) ->  
    verify_pp(PP(cons((left,d_right),pl)),x,hash(node(d_left,d_right)));  
  forall pl:list, x:bitstring, d_left,d_right:digest;  
    verify_pp(PP(pl),x,d_right) ->  
    verify_pp(PP(cons((right,d_left),pl)),x,hash(node(d_left,d_right)))  
  .
```

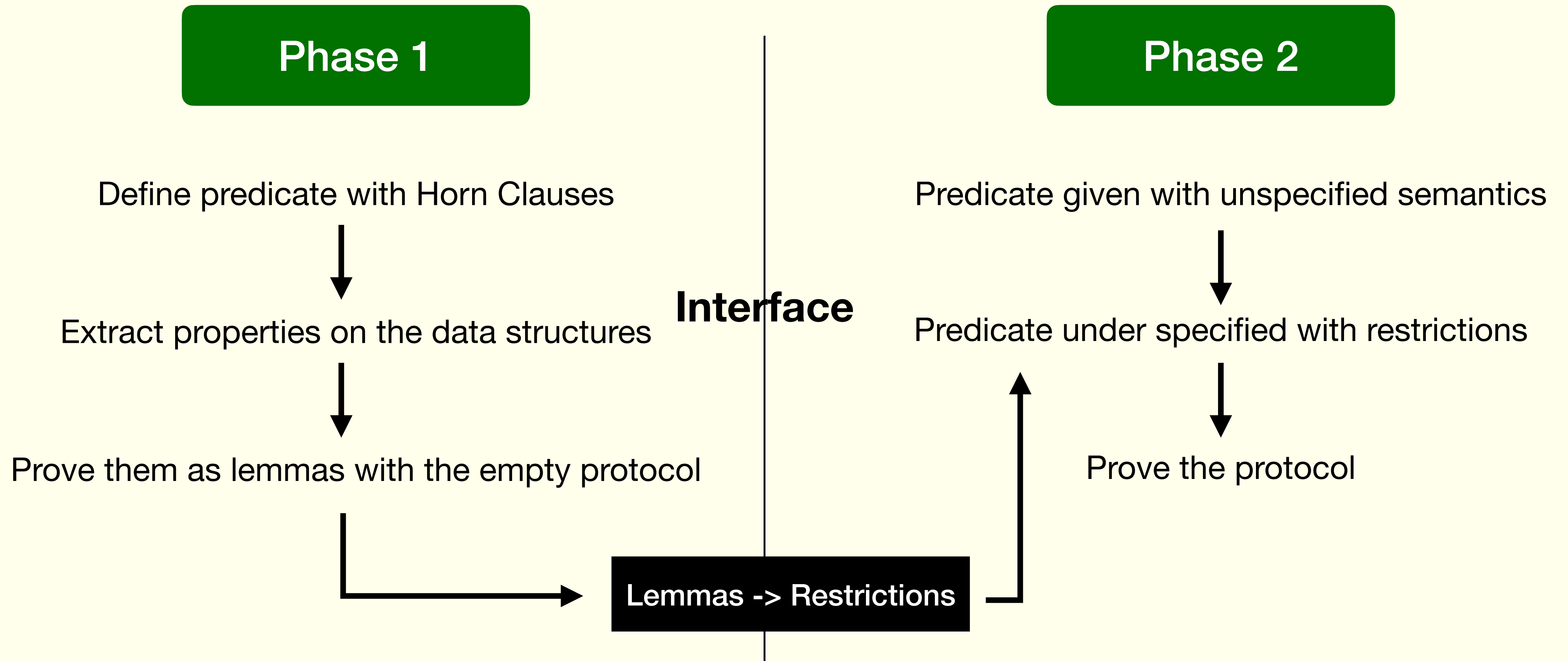
Will often not terminate if these Horn clauses are given with the protocol



# Properties on complex data structures [CMR-EuroS&P23]

Prove the protocol in two phases

Phase 2 proves the protocol for all implementations satisfying the interface



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# Properties on complex data structures [CMR-EuroS&P23]

## Extract of the interface

```
(* Transitivity of proof of extension *)  
lemma pe1,pe2,pe3:proof_of_extension, d1,d2,d3:digest;  
  verify_pe(pe1,d1,d2) && verify_pe(pe2,d2,d3) ==> verify_pe(pe3,d1,d3)  
.  
  
(* Proofs of presence are stable by proofs of extension *)  
lemma x:bitstring, pe:proof_of_extension, pp1,pp2:proof_of_presence,  
d1,d2:digest;  
  verify_pp(pp1,x,d1) && verify_pe(pe,d1,d2) ==> verify_pp(pp2,x,d2)  
.
```

---

# Properties on complex data structures

## Useful to model other predicates: `is_subterm`

In work with Véronique Cortier and Alexandre Debant on Election Verifiability

```
(* We do not implement the full subterm semantics but only a sufficient subset. *)
```

```
pred is_subterm(bitstring,bitstring).
```

```
clauses
```

```
  forall x,y:bitstring; is_subterm(x,hash((y,x)));
```

```
  forall x,y:bitstring; is_subterm(x,x);
```

```
  forall x,y,z:bitstring; is_subterm(x,y) -> is_subterm(x,hash((y,z)))
```

```
.
```

```
Lemma x,y:bitstring, uuid:election_id,j1,j2,i1,i2:nat,
```

```
h1,h2,balot1,balot2:bitstring;
```

```
  event(Balot_In_Bulletin_Board(uuid,j1,i1,balot1,h1)) &&
```

```
  event(Balot_In_Bulletin_Board(uuid,j2,i2,balot2,h2)) && i1 <= i2 ==>
```

```
  is_subterm(balot1,h2)
```

```
.
```

---

# Lemmas / Axioms / Restrictions in ProVerif

Does not allow temporal variables

**Allow to be more expressive in the order of events**

Does not allow disequalities and inequalities in the premises

**Allow to be more efficient in the application of the lemma**

Events in the conclusion do not restrict traces

**Avoid non-termination scenarios**

Conclusion cannot contain attacker, mess, table, or user defined predicate

**Improve precision**

Semantics of restrictions enforce that events in the conclusion occur before at least one fact of the premise.

**Allow to prove properties on complex data structure**

**Allow to prove liveness and accountability properties**

---

# Lemmas / Axioms / Restrictions in ProVerif (next release)

Includes [CMR-EuroS&P23]

Allow temporal variables

Allow to be more expressive in the order of events

Allow disequalities and inequalities in the premises

Allow to be more efficient in the application of the lemma

Events in the conclusion restrict traces

Avoid non-termination scenarios

Conclusion can contain any predicate

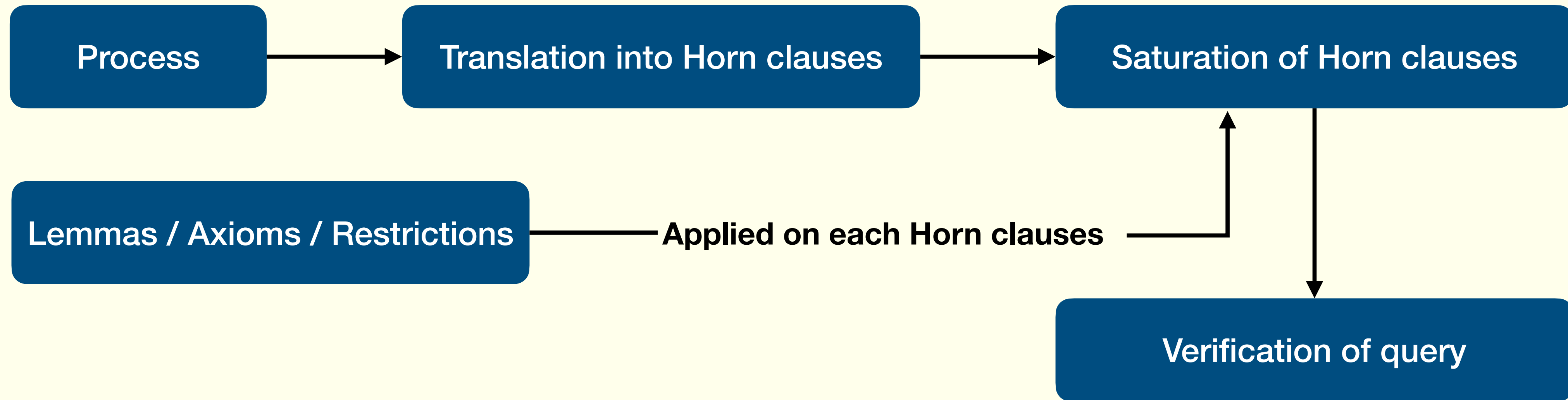
Improve precision

No semantics constraints on the occurrence order of events in restrictions.

Allow to prove properties on complex data structure

Allow to prove liveness and accountability properties

# Under the hood: any predicate in query conclusion



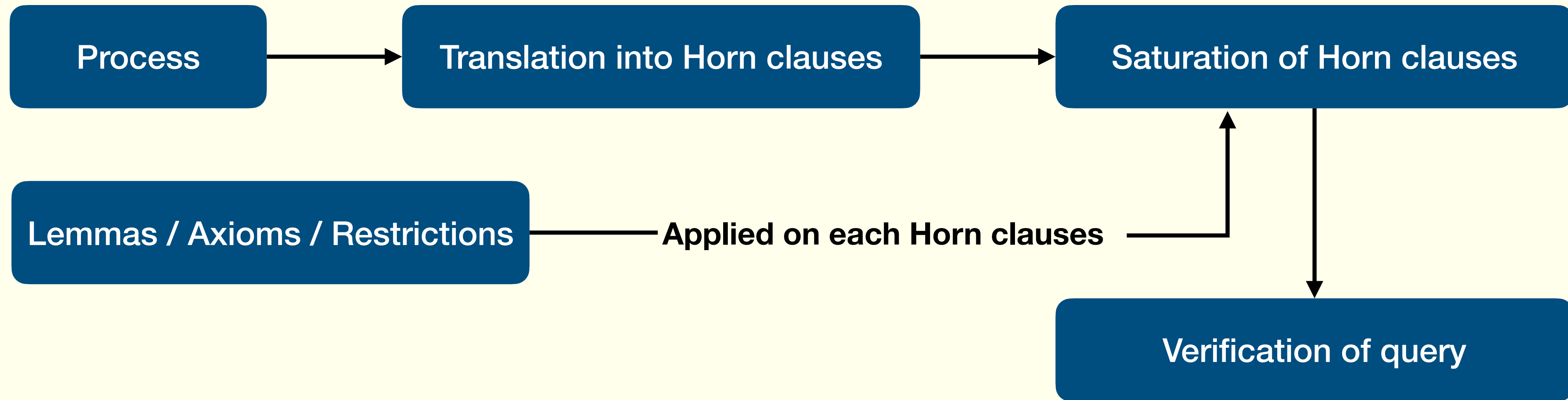
**Lemma**  $F_1 \wedge F_2 \Rightarrow G_1 \wedge G_2 \wedge G_3$

**Clause**  $H \rightarrow C$

**If there is a substitution  $\sigma$  such that  $F_1\sigma \wedge F_2\sigma \subseteq H$  then**

**$H \rightarrow C$  is replaced by  $H \wedge G_1\sigma \wedge G_2\sigma \wedge G_3\sigma \rightarrow C$**

# Under the hood: any predicate in query conclusion



**Lemma**  $F_1 \wedge F_2 \Rightarrow G_1 \wedge G_2 \wedge G_3$

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$H \rightarrow C$  is replaced by  $H \wedge G_1\sigma \wedge G_2\sigma \wedge G_3\sigma \rightarrow C$

Not always sound !

events



attacker facts





---

# Under the hood: any predicate in query conclusion

## Two predicates for events

*s*-event      *sure-event*: occurs only in hypotheses of Horn clauses

*m*-event      *may-event*: occurs only in conclusions of Horn clauses

**Consequence: facts with *s*-event predicates are never resolved !**

**Applying a lemma is sound if no added facts can be resolved.**



---

# Under the hood: any predicate in query conclusion

## Two predicates for events

*s*-event      *sure-event*: occurs only in hypotheses of Horn clauses  
*m*-event      *may-event*: occurs only in conclusions of Horn clauses

**Consequence: facts with *s*-event predicates are never resolved !**

**Applying a lemma is sound if no added facts can be resolved.**

For every predicate, we consider a *blocking* predicate that cannot be resolved

---

## Under the hood: any predicate in query conclusion

**Lemma**  $id:voter, v:vote; event(VoteCounted(id, v)) \implies attacker(v).$

**Clause**  $b\text{-event}(VoteCounted(id, C_1)) \wedge H \rightarrow att(C_1)$

---

## Under the hood: any predicate in query conclusion

**Lemma**  $id:voter, v:vote; \text{event}(\text{VoteCounted}(id, v)) \implies \text{attacker}(v).$

**Clause**  $\text{b-event}(\text{VoteCounted}(id, C_1)) \wedge H \rightarrow \text{att}(C_1)$

**After applying the lemma**

$\text{b-att}(C_1) \wedge \text{b-event}(\text{VoteCounted}(id, C_1)) \wedge H \rightarrow \text{att}(C_1)$

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**Lemma**  $id:voter, v:vote; event(VoteCounted(id, v)) \implies attacker(v).$

**Clause**  $b\text{-event}(VoteCounted(id, C_1)) \wedge H \rightarrow att(C_1)$

**After applying the lemma**

$b\text{-att}(C_1) \wedge b\text{-event}(VoteCounted(id, C_1)) \wedge H \rightarrow att(C_1)$

**Transformation rules are adapted to take blocking predicate into account**

The clause is removed by tautology

---

## Under the hood: forward restriction

```
process event Send | (event Goal; event Received).
```

```
query event(Send) ==> event(Goal).
```

False

---

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process event Send | (event Goal; event Received).
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True

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restriction event(Send) ==> event(Received).
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But ProVerif can't prove it... Why?

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process event Send | (event Goal; event Received).
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```
query event(Send) ==> event(Goal).
```

True

```
restriction event(Send) ==> event(Received).
```

**But ProVerif can't prove it... Why?**

**Clauses generated:**

→ event(Send)

b-event(Goal) → event(Received)

→ event(Goal)

## Under the hood: forward restriction

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process event Send | (event Goal; event Received).
```

```
query event(Send) ==> event(Goal).
```

True

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restriction event(Send) ==> event(Received).
```

But ProVerif can't prove it... Why?

Clauses after saturation:

$\text{b-event(Received)} \rightarrow \text{event(Send)}$

$\text{b-event(Goal)} \rightarrow \text{event(Received)}$

$\rightarrow \text{event(Goal)}$

Hypothesis of the clause cannot show the execution of event Goal.



# Under the hood: forward restriction

Idea: Two rounds of saturation

**restriction**  $\text{event}(\text{Send}) \implies \text{event}(\text{Received})$ .

$\text{b-event}(\text{Received}) \rightarrow \text{event}(\text{Send})$

becomes

$\text{b-event}(\text{Received}) \wedge \text{event}(\text{Received}) \rightarrow \text{event}(\text{Send})$

$\text{b-event}(\text{Received}) \wedge \text{b-event}(\text{Goal}) \rightarrow \text{event}(\text{Send})$

$\text{b-event}(\text{Goal}) \rightarrow \text{event}(\text{Received})$

$\rightarrow \text{event}(\text{Goal})$

Process

Translation into Horn clauses

1st Saturation of Horn clauses

Unblock events in hypothesis  
occurring in restrictions

2nd Saturation of Horn clauses

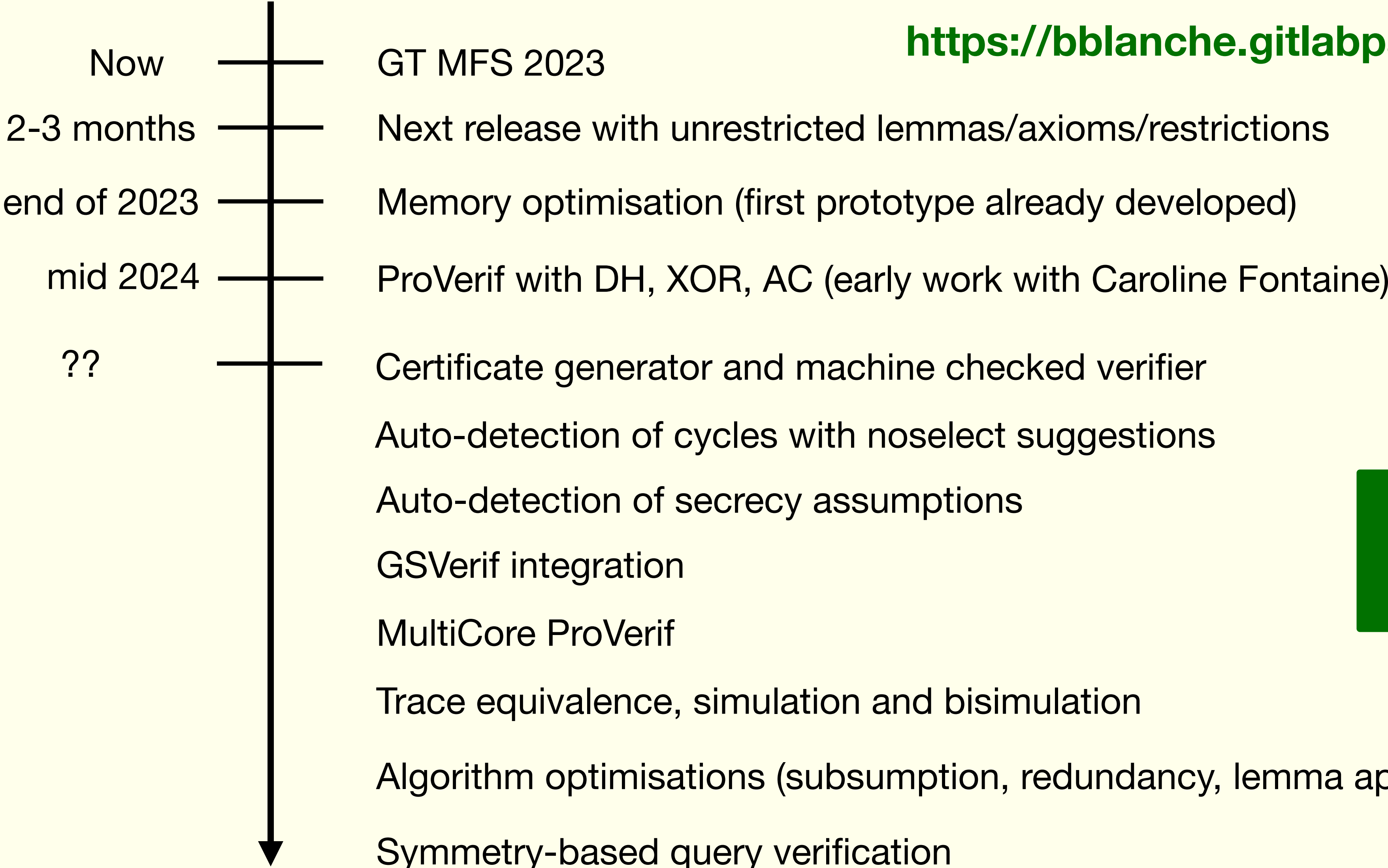
Verification of the query

# Timetable of ProVerif next releases

Available at

<https://gitlab.inria.fr/bblanche/proverif>

<https://bblanche.gitlabpages.inria.fr/proverif/>



**Interns wanted!**